# Chapter 22: Electrostatic potential
## and a trip down memory (PHY2048) lane...
## Thursday September 15th

- Electrostatic potential energy
    - Analogy with gravitation
    - Example problem
- Electrostatic potential
    - Relationship between $V$ and $E$
- Equipotential surfaces
    - Conductors
    - Relationship between $E$ and $V$

**Reading: up to page 380 in the text book (end Ch. 22)**

# Electrostatic Potential Energy

***Consider the analogy with gravitation:***

$$\vec{\mathbf{F}}_E = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} \qquad \vec{\mathbf{F}}_G = -G\frac{m_1 m_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

***In the gravitational case, we defined differences in potential energy in the following way:***

$$\Delta U = U_f - U_i = -W_{if} = -\int_{r_i}^{r_f}\vec{\mathbf{F}}\cdot d\vec{r} = -Gm_1 m_2\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

Recall: if you raise a book through a height $h$, you do positive work because your force and the book's displacement are parallel; however, the gravitational field does negative work. Thus, the increase in $U$ is equal the negative of the work done by the field, $\Delta U = -W_g$.

# Electrostatic Potential Energy

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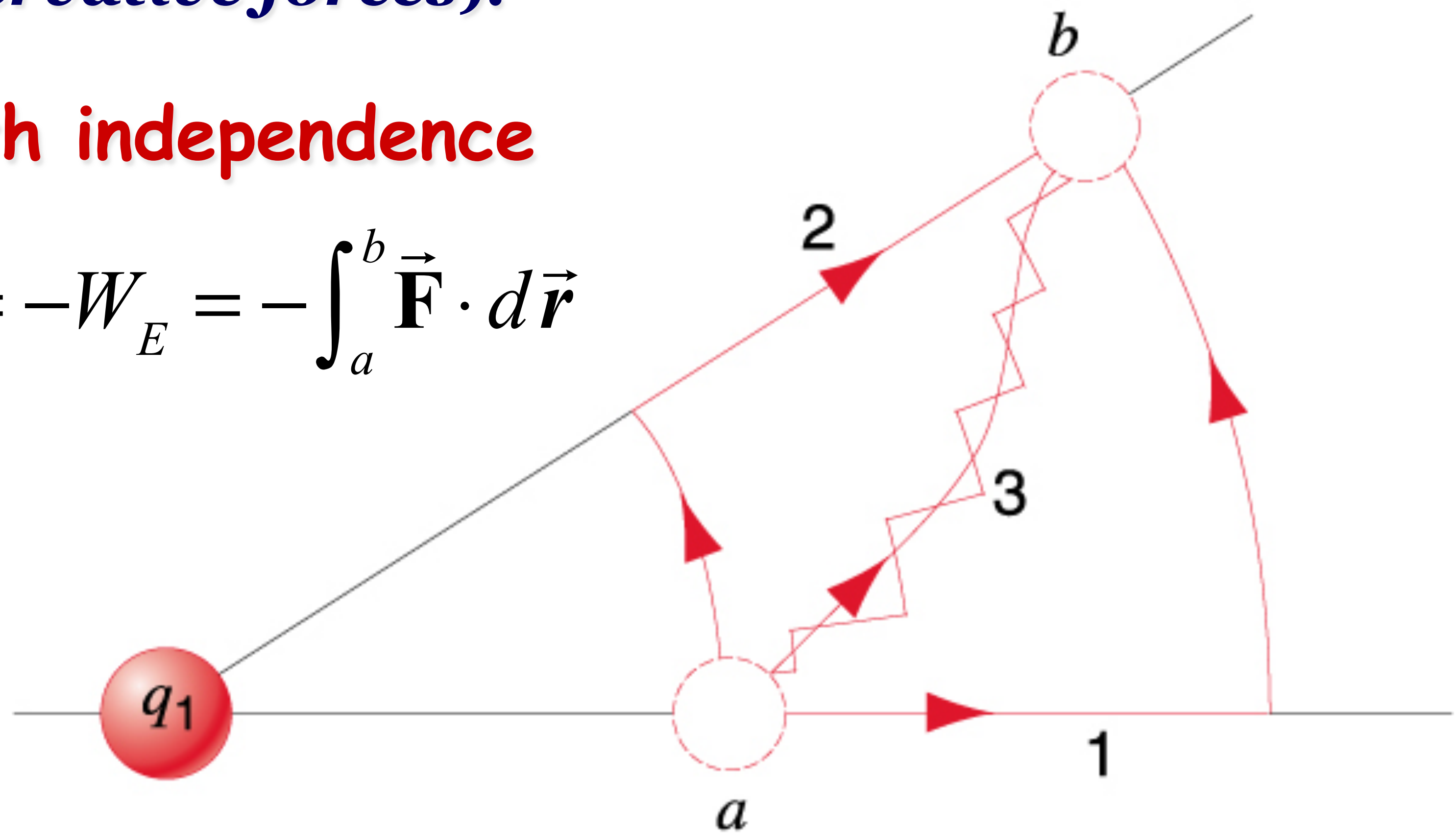
***You may have been told that gravity is a conservative force. Thus, the above result does not depend on the path taken between $r_i$ and $r_f$.***

# Electrostatic Potential Energy

- ***Thus, it should not surprise you that the electrostatic force is conservative also.***
- ***It is this property that allows us to define a scalar potential energy (one cannot do this for non conservative forces).***

## Path independence

$$\Delta U = -W_E = -\int_a^b \vec{\mathbf{F}} \cdot d\vec{\boldsymbol{r}}$$



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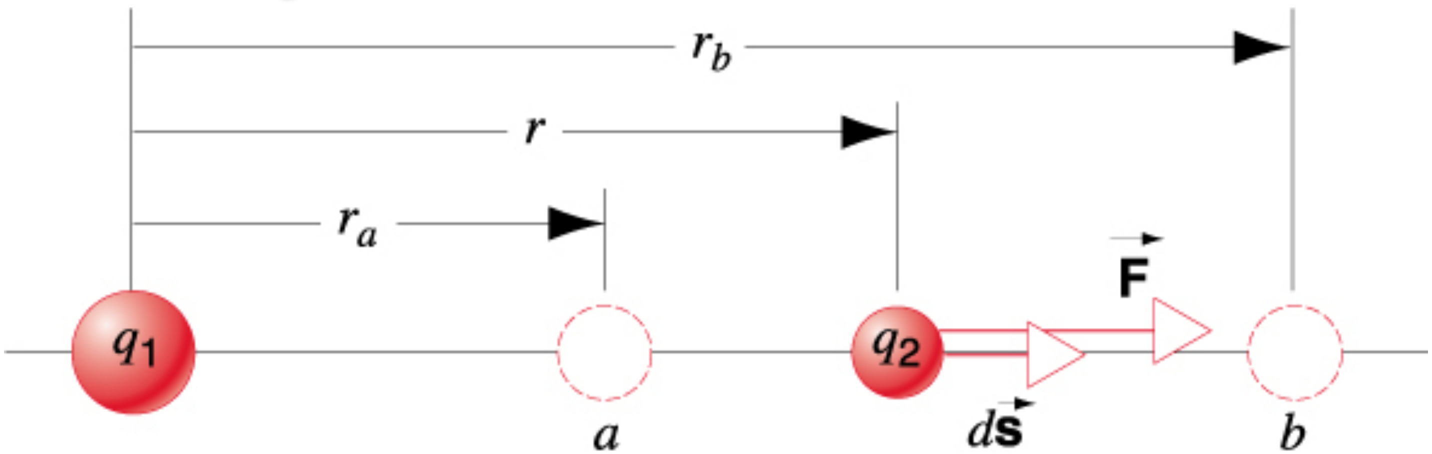


$$\Delta U = -\int_a^b \vec{\mathbf{F}} \cdot d\vec{\boldsymbol{r}} = +\frac{1}{4\pi\varepsilon_o} q_1 q_2 \left( \frac{1}{r_b} - \frac{1}{r_a} \right)$$

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**The sign is not a problem. It is taken care of by the signs of the charges $q_1$ and $q_2$.**

# Electrostatic Potential Energy

- ***The potential energy is a property of both of the charges, not one or the other.***
- ***If we choose a reference such that $U = 0$ when the charges are infinitely far apart, then we can simplify the expression for the potential energy as follows.***

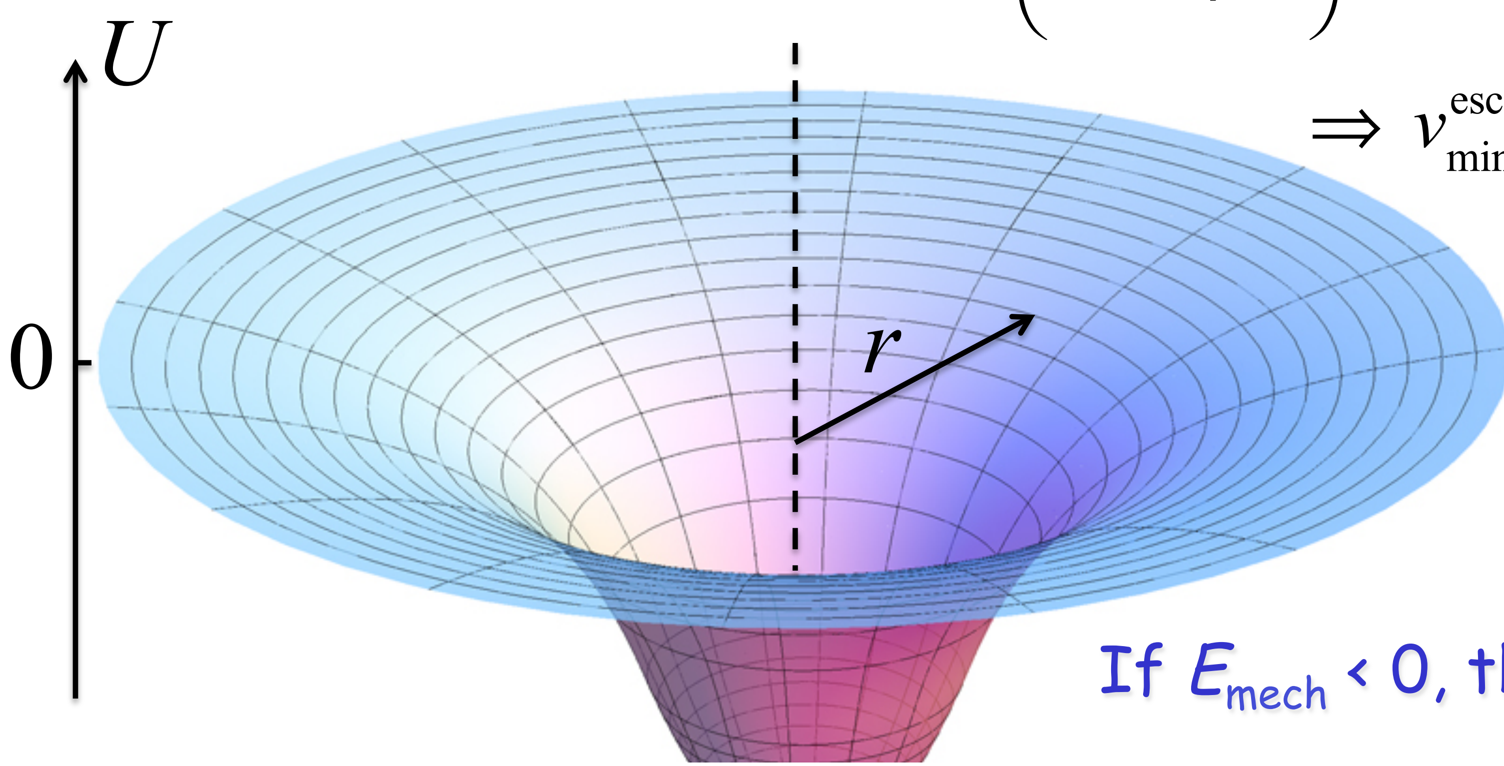
$$U(r) = -\int_{\infty}^{r} \vec{\mathbf{F}} \cdot d\vec{\boldsymbol{r}} = +\frac{1}{4\pi\varepsilon_o} q_1 q_2 \left( \frac{1}{r} - \frac{1}{\infty} \right) = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r}$$

**Again, the sign of $U$ is not a problem. It is taken care of by the signs of the charges $q_1$ and $q_2$.**

# Escape speed

- In order for a negative charge $q$ with mass $m$ to escape the electrostatic pull of a positive charge $Q$, the kinetic energy, $K$, must be sufficient to overcome the negative electrostatic potential energy, $U$, i.e., the total mechanical energy must be greater than zero.

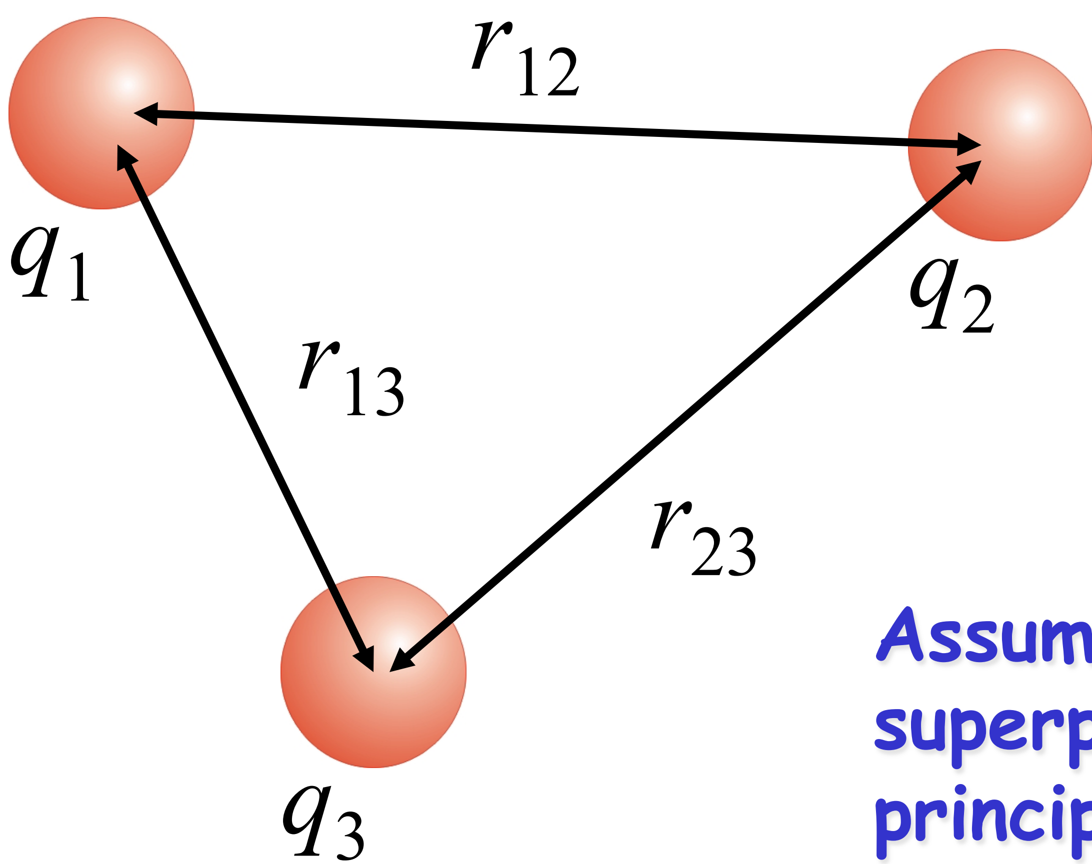
$$E_{mech} = K + U = \tfrac{1}{2}mv^2\left(-k\frac{|Q||q|}{r}\right) > 0$$




$$\Rightarrow v_{\min}^{\text{escape}} = \sqrt{\frac{2kQq}{mr}}$$

If $E_{mech} < 0$, then trapped.

# Potential Energy of a System of Charges

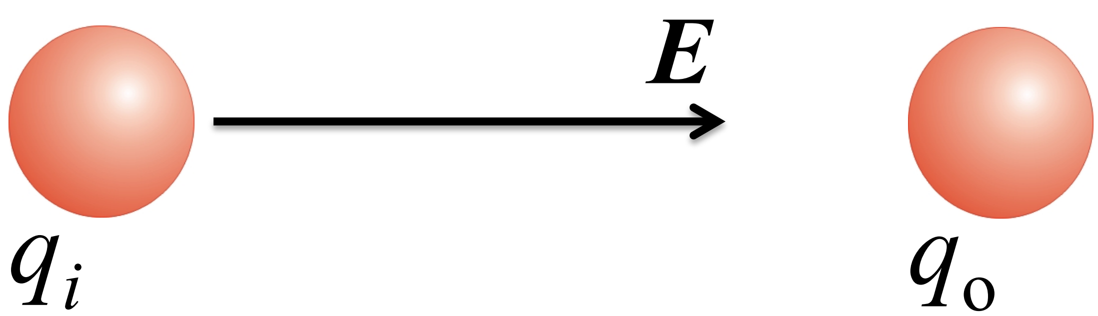



Assumes that the superposition principle is valid

$$U = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_o}\frac{q_2 q_3}{r_{23}}$$

# The Electrostatic Potential

- *Suppose we take a test charge $q_o$ and move it from point a to point b under the influence of the electrostatic force due to a system of source charges $q_i$.*
- *We can eliminate the test charge $q_o$ from the problem in exactly the same fashion as we did in chapter 20 when defining the electric field. In doing so, we define a new quantity known as the Electrostatic Potential difference $\Delta V$:*



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$$\Delta V = V_b - V_a = \frac{U_b - U_a}{q_o} = -\frac{W}{q_o} = -\int_a^b \frac{\vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}}{q_0}$$

- ***This 'scalar potential' depends only on the details of the static charge distribution.***
- ***Note the units: Joules per Coulomb (J/C), also known as Volts (V).***

# The Electrostatic Potential

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Recall: $\vec{\mathbf{F}} = q_o \vec{\mathbf{E}}$

$$\Rightarrow \Delta V = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{r}} = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

- ***This 'scalar potential' depends only on the details of the source charge distribution* (in this case, $q$).**

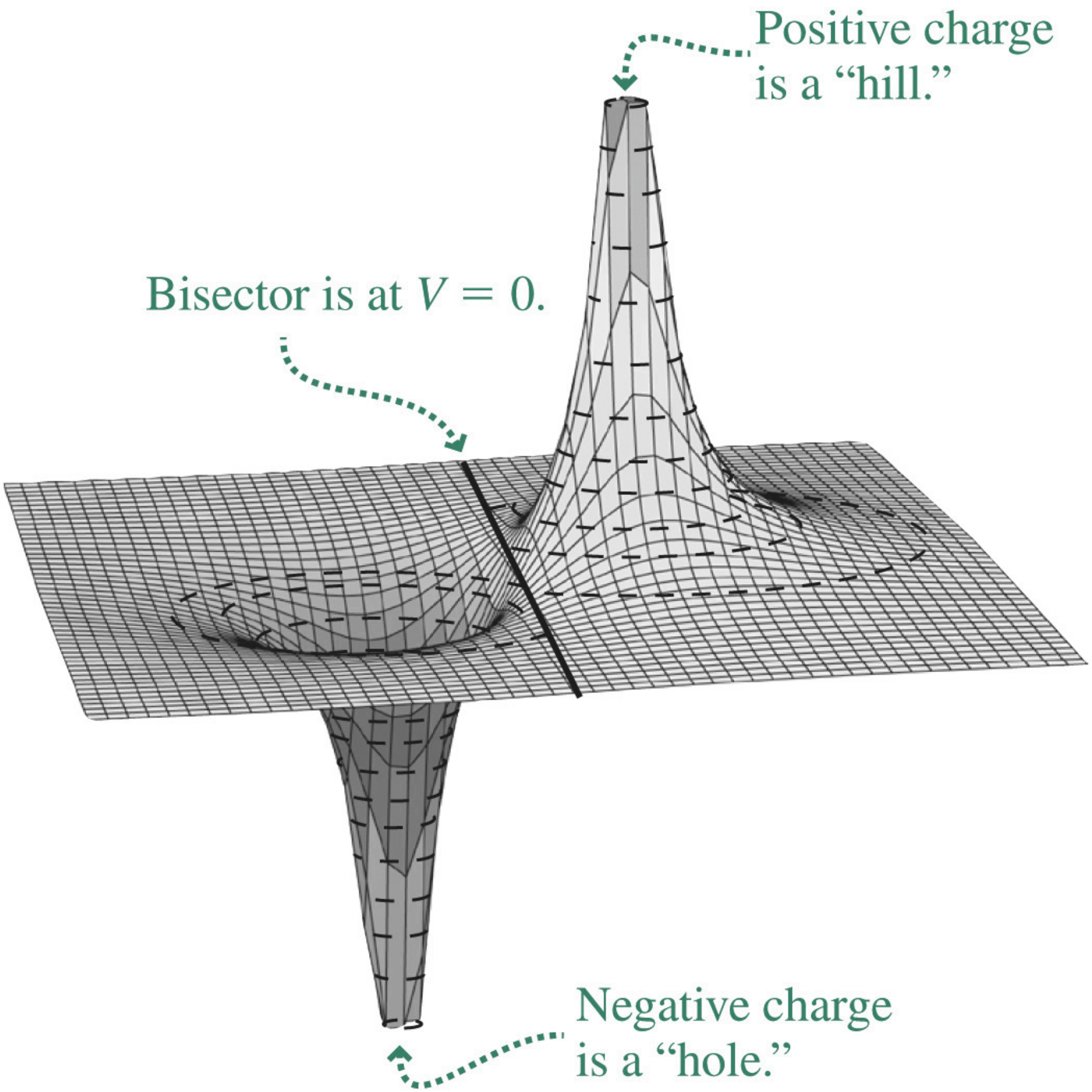
# The Electrostatic Potential

- *Just as in the case of potential energy, we may choose a reference which is infinitely far from the source charges, and set the absolute potential to be zero at this point in space.*
- *In doing so:*

$$V(r) = -\int_{\infty}^{r} \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{r}} = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r} - \frac{1}{\infty}\right) = \frac{q}{4\pi\varepsilon_o}\frac{1}{r} = k\frac{q}{r}$$

- *The absolute value of the potential is not important, i.e. we could have decided to set the potential to 100V at our reference. As we shall see, it is only potential differences that really matter.*
- *In fact, here on earth we choose the ground as our $V = 0$ reference.*

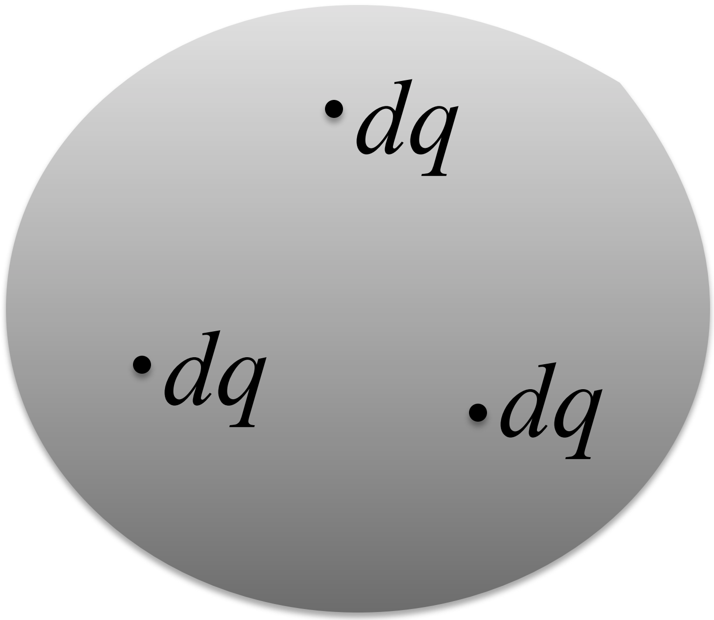
# Potential due to a dipole



# The main subject of this course

## Superposition principle still holds

Source charges




$$V = \frac{1}{4\pi\varepsilon_o}\int \frac{dq}{r}$$

Test charge

$\bullet Q$

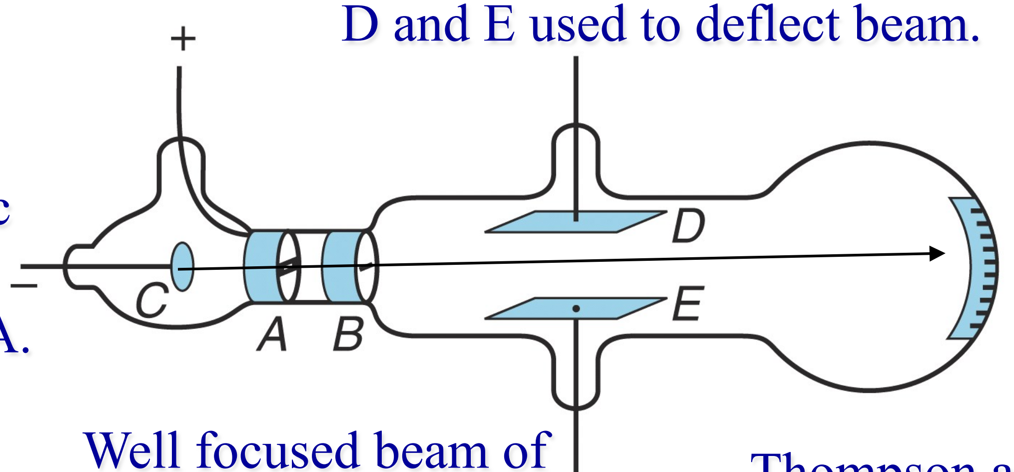
- This problem is considerably simpler when dealing with a scalar potential, as opposed to a vector field, i.e., the electric field.
- We shall also see that it is sometimes easier to first calculate *V*, then use *V* to find *E*.

# Discovery of the electron: J. J. Thompson (1897)

## Cathode ray tube - or CRT (Nobel prize in 1906)

Charged particles evaporate from heated cathode C. Negative charges accelerate in electric field between the cathode and anode A.

Electric field between D and E used to deflect beam.




Phosphorescent screen used to detect the cathode ray.

Well focused beam of negative particles passes through the slit B.

Thompson also applied a magnetic field into the page, which also deflected the ray.

**Basis for so much technology: TV, oscilloscope, mass spectrometer, etc...**

$$F = qE = ma \qquad \Rightarrow \qquad a = \frac{q}{m}E$$

In order to measure $q/m$ it is essential to know the velocity of the beam.

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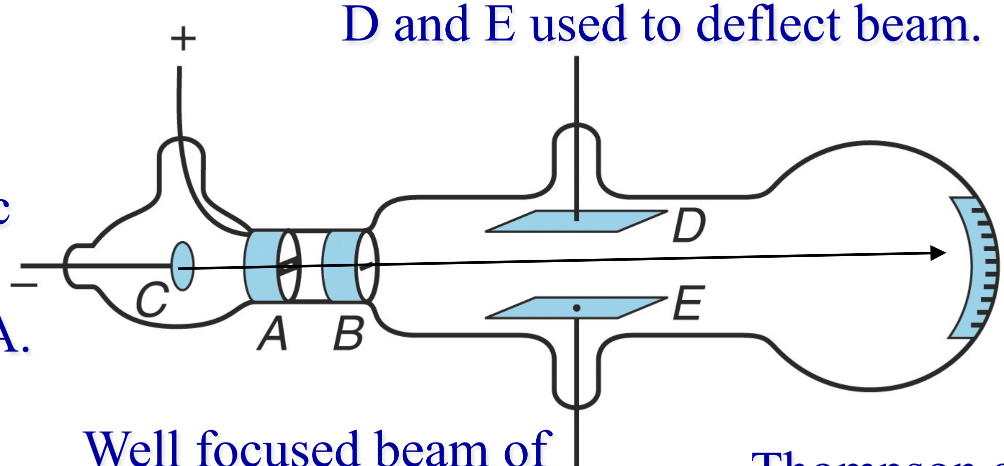


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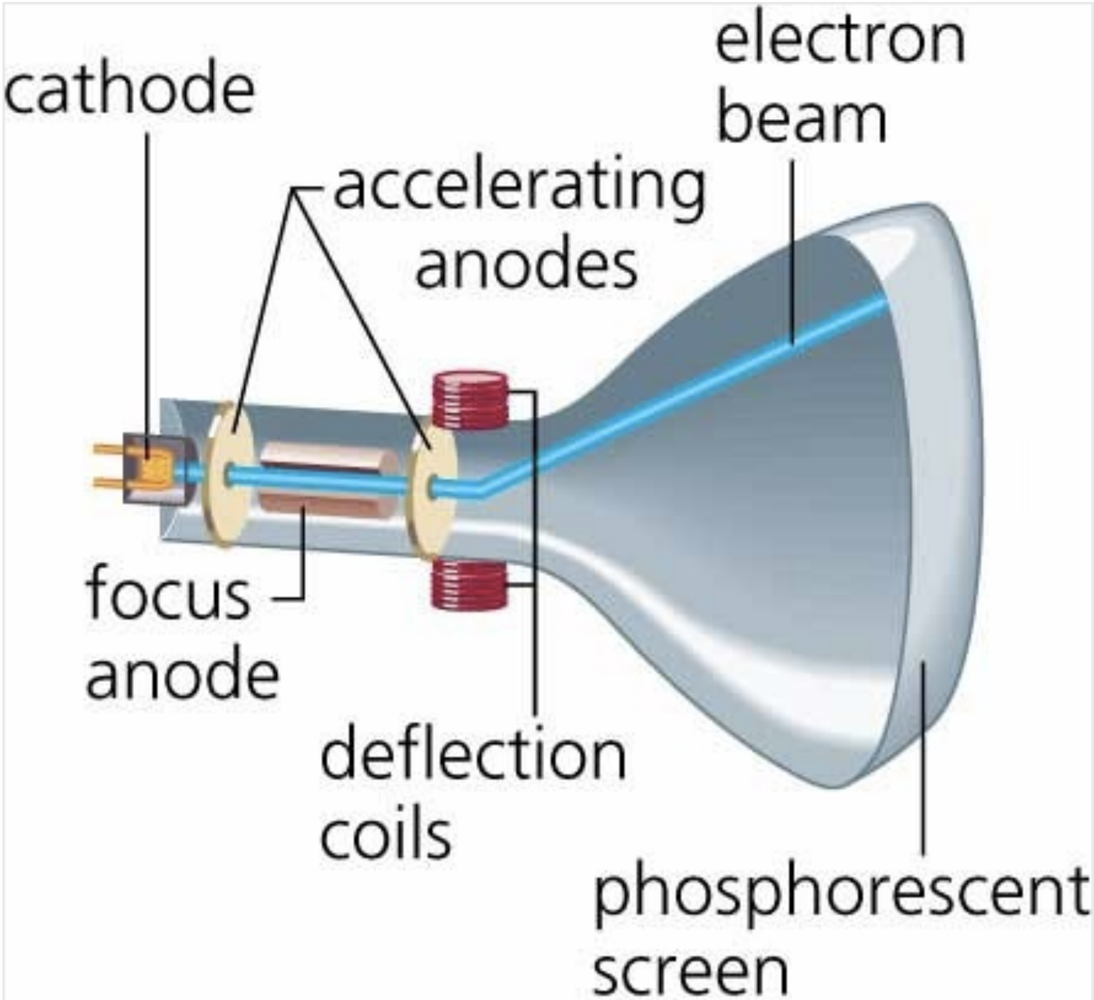
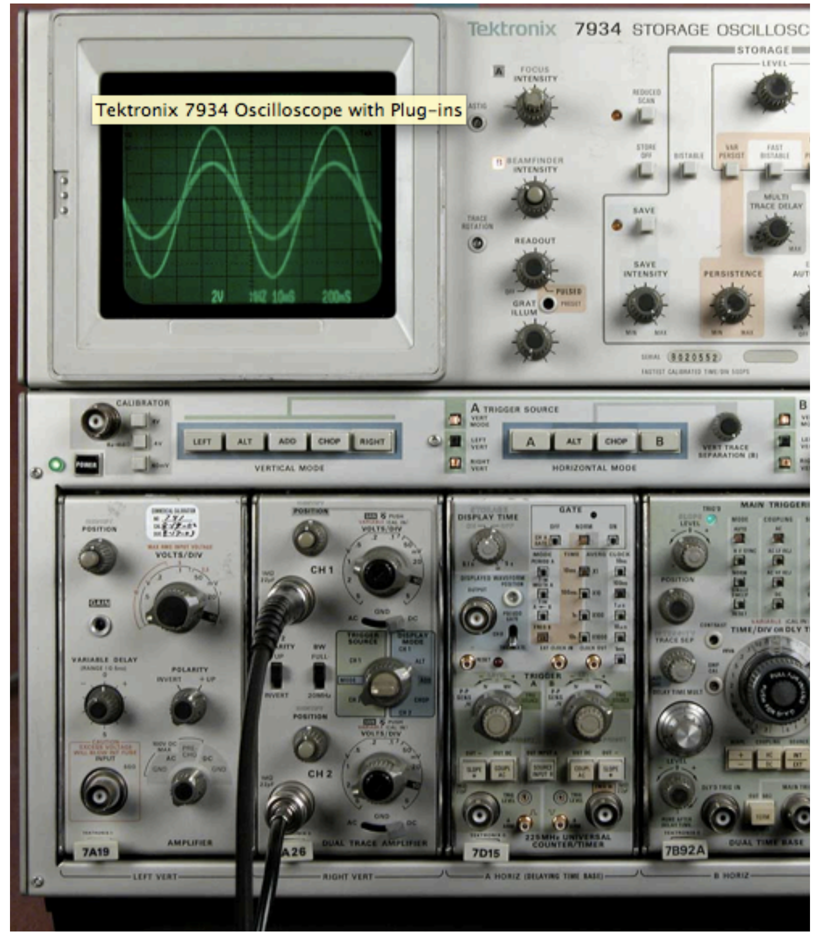
$$\Delta U = q\Delta V = -\Delta K \quad \Rightarrow \quad v = \sqrt{\frac{2|q||\Delta V|}{m}}$$

$K$ often quoted in electron-volts (eV): 1 eV = $e\times(1\text{ V}) = 1.6\times10^{-19}$ J

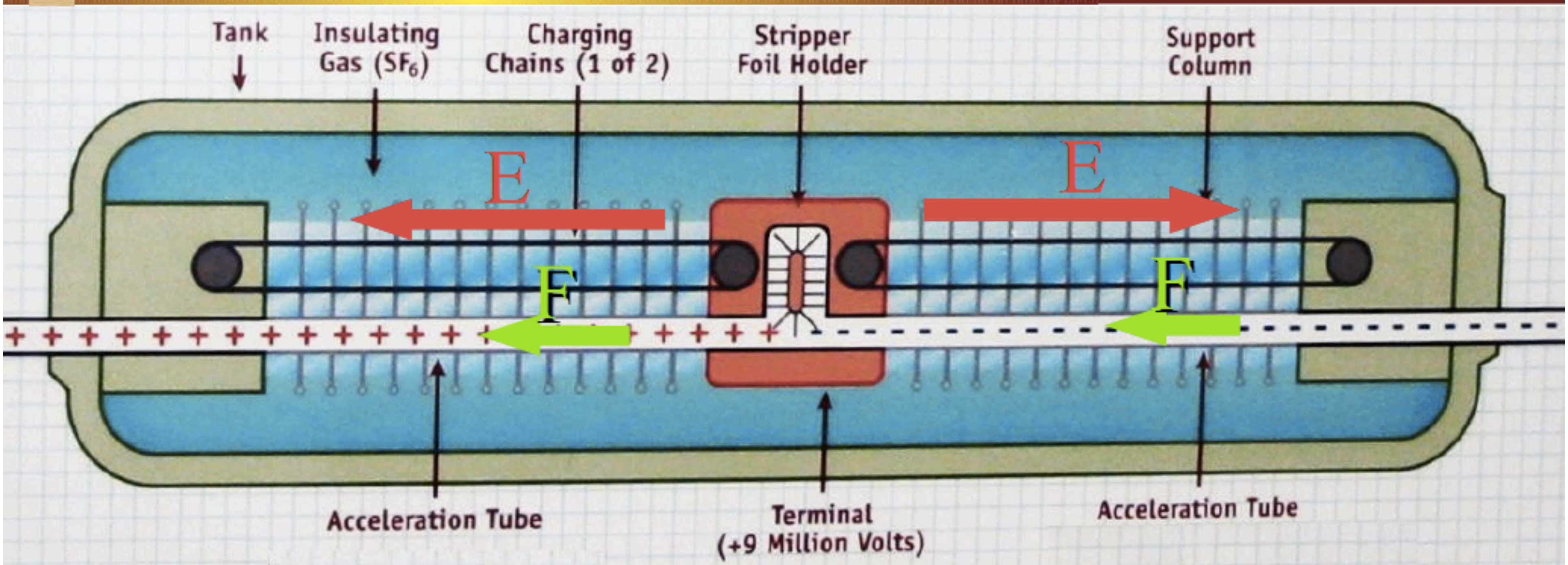
# Cathode Ray Tubes (CRTs)




Lab #6

# Electric Fields - Accelerators




- Tandem Van-de-Graaf accelerator E=1.8 MV / m (Here at FSU, right across the street)
- Takes negative ions, attracted by +9 Million Volt
- In the center (at +9 Million Volts), it converts them to positive ions which are then repelled